

# Image Fusion Based on Fast and Adaptive Bidimensional Empirical Mode Decomposition

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**Abstract** – *The recently introduced Fast and Adaptive Bidimensional Empirical Mode Decomposition (FABEMD) method is evaluated in image fusion applications. The FABEMD method does not suffer from the problems related to the traditional two dimensional scattered data interpolation based EMD method. As a result, the quality of the extracted Bidimensional Intrinsic Mode Functions (BIMFs) are better and the whole process is less time-consuming, a desirable feature for real-world image fusion. Besides, the ability to generate the same number of BIMFs with matching scales and the structure recognition capability facilitate heterogeneous applications. Simulation results demonstrate the effectiveness of the proposed approach in the context of multi-focus image fusion.*

**Keywords:** Image fusion, Bidimensional Empirical Mode Decomposition (BEMD), Order-statistics filter.

## 1 Introduction

Empirical Mode Decomposition (EMD), first developed by Huang et al. [1], is a powerful tool for decomposing nonlinear and nonstationary signals. The bases of this AM-FM decomposition, called the Intrinsic Mode Functions (IMFs), are generated in an adaptive way. As a fully data driven method, EMD has found widespread application in one-dimensional signal processing. Recently, this decomposition technique has also been extended to analyze two dimensional (2D) images, which is known as Bidimensional EMD (BEMD) [2].

Both EMD and BEMD method depend on a process called the sifting [1] where the local extrema points are initially detected and subsequent interpolation of those points is carried out to find the envelope. In each iteration the mean envelope is then constructed and subtracted from the original signal to obtain the candidate IMFs. From the candidate IMFs, actual IMFs are repeatedly extracted based on some well-defined IMF characteristics and the process is repeated until all the IMFs are generated. The effect of sifting is to eliminate

the riding waves, i.e., to make the number of extrema equal to that of zero-crossings as well as to make the envelopes symmetric and thus the amplitudes of the oscillations more even.

Traditional BEMD [2] which depends upon two dimensional scattered data interpolation method suffers from lack of interpolation centers at the boundary region of the extrema map and very few arbitrarily distributed extrema points at some stages of the sifting process. As a result, it often produces incorrect Bidimensional IMFs (BIMFs). As the surface interpolation method is an iterative optimization process to fit the points into a surface, the BEMD process is excessively time consuming. As a result, it requires an additional stopping criterion, resulting in inaccurate and incomplete decomposition. Overshooting or undershooting is another problem of interpolation-based envelope estimation, which causes inaccurate BIMFs [3].

To overcome these limitations of BEMD, a novel approach called Fast and Adaptive BEMD (FABEMD) was proposed recently by Bhuiyan et al. [3, 4]. It substitutes the 2D scattered data interpolation step of BEMD by a direct envelope estimation method. In this technique, spatial domain sliding order-statistics filters, namely, MAX and MIN filters, are utilized to obtain the running maxima and running minima of the data. Application of smoothing operation to the running maxima and minima results in the desired upper and lower envelopes respectively. The size of the order-statistics filters is derived from the available information of maxima and minima maps.

Recently, EMD has been employed in for data fusion, within the so-called “information fusion via fission” framework [5], whereby only the “relevant” IMFs are recombined into a restored signal. Looney and Mandic pointed out in [6] the following problems that have to be addressed to use EMD for data fusion:

- The number of IMFs from each source must be equal.
- The IMFs from each source should also be matched

in frequency scale to enable a meaningful comparison.

To this end, they proposed complex extensions [7, 8, 9] of the EMD algorithm to decompose data from heterogeneous sources simultaneously [6]. Although this allowed the fusion of two partially defocused images successful, due to the 1D nature of the complex EMD which ignores the correlation of 2D image, it has no structure recognition capability. On the other hand, though BEMD has structure recognition capability, it does not provide the same number of BIMFs for two separate images unless the algorithm is forcefully stopped after a given number of BIMFs have been generated. However, the FABEMD algorithm has the structure recognition capability and it generates same number of BIMFs from separate sources and BIMFs properties from each source are matched well.

In this paper, we propose to use the recently introduced FABEMD method to perform image fusion based on the common spatio-temporal scales. To that cause, firstly the filter bank structure of the FABEMD method is confirmed by applying it to the bidimensional Gaussian white noises. Then the FABEMD method is tested on sythetic texture images to validate our claim that the decompostions of different sources by FABEMD are matched in number and frequency scale and lastly it is applied to fuse two partially defocused real images to show the potential of the method.

## 2 FABEMD Overview

Let the original image be denoted by  $S(m, n)$ , the  $i$ -th BIMF as  $I_i(m, n)$ , and the residue as  $R(m, n)$ . In the decomposition process,  $I_i(m, n)$  is obtained from its source image  $J_i(m, n)$ , where  $J_i(m, n) = J_{i-1}(m, n) - I_{i-1}(m, n)$  and  $J_1(m, n) = S(m, n)$ . The steps of the BEMD process can be explained as follows [3]:

1. Set  $i = 1$  and  $J_i(m, n) = S(m, n)$ .
2. Obtain the local maxima and minima maps of  $J_i(m, n)$ , denoted as  $M_i(m, n)$  and  $N_i(m, n)$  respectively using the neighboring window method. In this method, a data point is considered as a local maximum (minimum), if its value is strictly higher (lower) than all of its neighbors within a window. Generally, a  $3 \times 3$  window results in optimum extrema maps for a given 2D data.
3. Calculate the size of the order-statistics filters and smoothing averaging filters from  $M_i(m, n)$  and  $N_i(m, n)$  as discussed later in this section.
4. Form the upper envelope  $U_{Ei}(m, n)$  and lower envelope  $L_{Ei}(m, n)$  from  $J_i(m, n)$  using the order-statistics filter previously described and then form smoothed upper envelope  $U_{ESi}(m, n)$  and smoothed lower envelope  $L_{ESi}(m, n)$  using

smoothing averaging filters as discussed later in this section.

5. Find the mean/average envelope ( $M_{Ei}$ ) as  $M_{Ei}(m, n) = (U_{ESi}(m, n) + L_{ESi}(m, n))/2$ .
6. Calculate  $I_i(m, n) = J_i(m, n) - M_{Ei}(m, n)$ .
7. Set  $J_{i+1}(m, n) = J_i(m, n) - I_i(m, n)$ .
8. If the source image  $J_{i+1}(m, n)$  has less than three extrema points, set  $R(m, n) = J_{i+1}(m, n)$  and the decomposition is complete. Otherwise, set  $i = i + 1$  and go to step (2) and continue to step (8).

FABEMD utilizes two order statistics filters to approximate extrema envelopes, where a MAX filter is applied to the upper envelope and a MIN filter to the lower envelope. The sizes of these filters are determined based on  $M_i(m, n)$  and  $N_i(m, n)$  defined earlier. For each local *maximum* (*minimum*) point in  $M_i(m, n)$  ( $N_i(m, n)$ ), the Euclidean distance to the nearest other local *maximum* (*minimum*) point is computed and stored in an array, denoted as  $d_{adj-max}$  ( $d_{adj-min}$ ), where the number of elements is equal to the number of local *maxima* (*minima*) points in the *maxima* (*minima*) map  $M_i(m, n)$  ( $N_i(m, n)$ ). Considering the square window, the gross window width  $w_{en-g}$  for order statistics filters can be selected in different ways using the distance values in  $d_{adj-max}$  and  $d_{adj-min}$  among which two choices are considered here as given below [3]:

$$w_{en-g} = \min\{\min\{d_{adj-max}\}, \min\{d_{adj-min}\}\} \quad (1)$$

$$w_{en-g} = \max\{\max\{d_{adj-max}\}, \max\{d_{adj-min}\}\} \quad (2)$$

where, the operator  $\max\{\cdot\}$  denotes the maximum value of the elements in the array and  $\min\{\cdot\}$  denotes the minimum value of the elements in the array.  $w_{en-g}$  is rounded to the nearest odd integer to get the final window width  $w_{en}$ . In [3], the Order Statistics Filter Widths (OSFWs) obtained via Eqs. (1) and (2) are defined as Lowest Distance OSFW (LD-OSFW) and Highest Distance OSFW (HD-OSFW) respectively. The choice of  $w_{en}$  from the above options depends on the application and/or desired BIMF characteristics.

With the determination of window size  $w_{en}$ , MAX and MIN filters are applied to the corresponding  $J_i(m, n)$  to obtain the upper and lower envelopes,  $U_{Ei}$  and  $L_{Ei}$ , as specified below [3]:

$$U_{Ei}(m, n) = \max_{(s,t) \in Z_{mn}} J_i(s, t) \quad (3)$$

$$L_{Ei}(m, n) = \min_{(s,t) \in Z_{mn}} J_i(s, t) \quad (4)$$

where the value of the upper envelope  $U_{Ei}$  at any point  $(m, n)$  is simply the maximum value of the elements in  $J_i(m, n)$  in the region defined by  $Z_{mn}$ , where  $Z_{mn}$  is the square region of size  $w_{en} \times w_{en}$  centered at any point  $(m, n)$  of  $J_i(m, n)$ . Similarly, the value of the

lower envelope  $L_{Ei}$  at any point  $(m, n)$  is simply the minimum value of the elements in  $J_i(m, n)$  in the region defined by  $Z_{mn}$ .

To obtain smooth continuous surfaces for upper and lower envelopes, averaging smoothing operations are carried out on both  $U_{Ei}(m, n)$  and  $L_{Ei}(m, n)$ , which may be expressed as below:

$$U_{ESi}(m, n) = \frac{1}{w_{sm} \times w_{sm}} \sum_{(s,t) \in Z_{mn}} U_{Ei}(s, t) \quad (5)$$

$$L_{ESi}(m, n) = \frac{1}{w_{sm} \times w_{sm}} \sum_{(s,t) \in Z_{mn}} L_{Ei}(s, t) \quad (6)$$

where  $Z_{mn}$  is the square region of size  $w_{sm} \times w_{sm}$  centered at any point  $(m, n)$  of  $U_{Ei}$  or  $L_{Ei}$ ,  $w_{sm}$  is the window width of the averaging smoothing filter and  $w_{sm} = w_{en}$ .

### 3 FABEMD as a filter bank

The filter bank structure of EMD has been described by Flandrin *et al.* in [10, 11]. Similarly, Damerval *et al.* also analyzed the filter bank structure of BEMD in [12]. In Figure 1, it is shown that the FABEMD method also creates a selective filter bank when applied to bidimensional Gaussian white noises. For that cause, the average Fourier transforms of BIMFs over 200 realizations of a bidimensional  $128 \times 128$  Gaussian white noise with  $\sigma^2 = 1$  are computed. The results of the average Fourier transforms are displayed in Figure 1 for the BIMFs and the residue. As expected, the first modes contain the highest frequencies, while the other modes contain the lower frequencies.

### 4 Image fusion

As mentioned earlier, the FABEMD algorithm has the structure recognition capability and it generates the same number of BIMFs from separate sources and BIMFs properties from each source are matched well. To guarantee that BIMFs from the images being analysed are matched in number and properties i.e., to find a set of common scales which is the necessary condition for fusion, we choose the size of the order statistics filter used to estimate the envelopes as one of the follows:

$$W_{en-g} = \min\{\min\{d_{adj-max-i}\}, \min\{d_{adj-min-i}\}\} \quad (7)$$

$$W_{en-g} = \max\{\max\{d_{adj-max-i}\}, \max\{d_{adj-min-i}\}\} \quad (8)$$

where  $d_{adj-max-i}$  is same as  $d_{adj-max}$  defined earlier but now for the  $i$ -th image used where  $i = 1, 2, \dots, N$  denotes the number of images used for fusion.

To validate our claim, two synthetic texture images were constructed from a set of four sinusoidal synthetic texture components of high, medium, low and very low

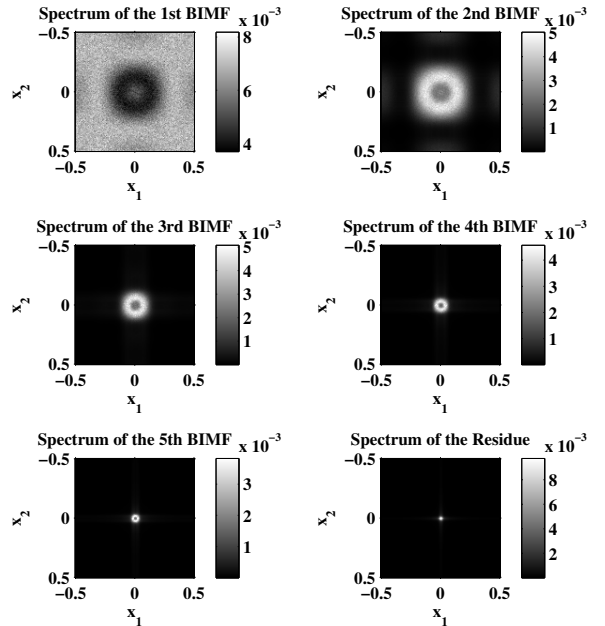


Figure 1: Average Fourier transform of BIMFs obtained over 200 realizations of a  $128 \times 128$  bidimensional Gaussian white noise with  $\sigma^2 = 1$

frequency. The high frequency (HF) and very low frequency (VLF) component were common to both the images whereas the first image additionally had the medium frequency (MF) component and the second image had the low frequency (LF) component. Figure 2 presents the two images used for this test.

First, the complex extension of EMD is used as described in [6] for the decomposition of the test images which is shown in Figure 3. Though this method provides the same number of IMFs for both the images (14 IMFs but only first 4 IMFs and the residues are shown for simplicity) together with matching scales, the method could not recognize the structure of the image. Moreover, the artifacts in the IMFs due to the 1D nature of the algorithm (as it does not consider the correlation of the 2D image) is seen in the third and fourth IMF of the two images of Figure 3.

Next we applied the BEMD used in [2] to decompose the test images which was forcefully stopped after obtaining four BIMFs for both the images. The result is depicted in Figure 4. From the figure, it is seen that though the algorithm exhibits the structure recognition capability, it does not provide matching scales. The 2nd BIMF of the first image corresponds to the medium frequency component whereas the 2nd BIMF of the second image corresponds to the low frequency component. Besides, the algorithm is forcefully stopped here to generate the same number of BIMFs for both the images.

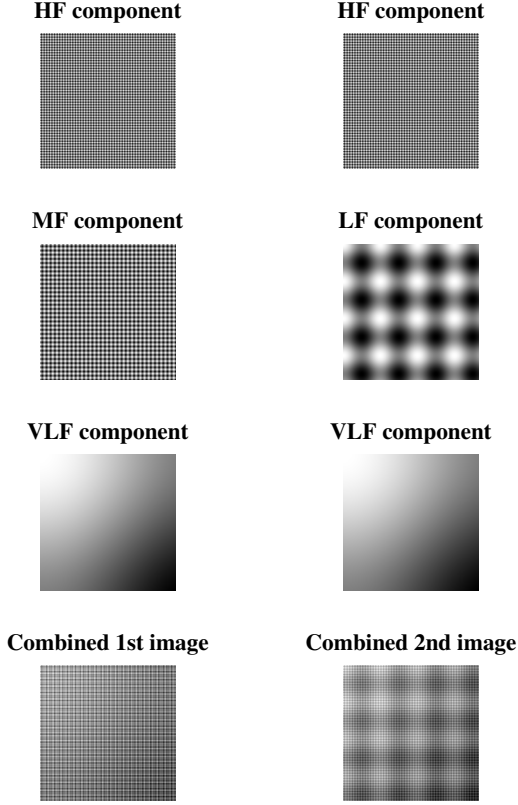


Figure 2: Two synthetic texture image used for the test

Another point to be noted, the artifacts due to lack of extremas in the boundary region propagates into the mid region for the later modes of decomposition is also seen in the Figure 4.

Finally, the FABEMD method has been applied to the test images. The result of the decomposition is shown in Figure 5. From the figure, we can see that 2nd BIMF of the first image captures the medium frequency component whereas this frequency is absent in the 2nd BIMF of the second image. Similarly, the 3rd BIMF of the second image captures the low frequency component whereas the 3rd BIMF of the first image captures nothing as this frequency is absent in the original image. As a result, it is clearly verified that FABEMD not only has the structure recognition capability but it also provides the same number of BIMFs from each source with matching scales.

Now using FABEMD, two partially defocused real images A and B are fused with the following fusion

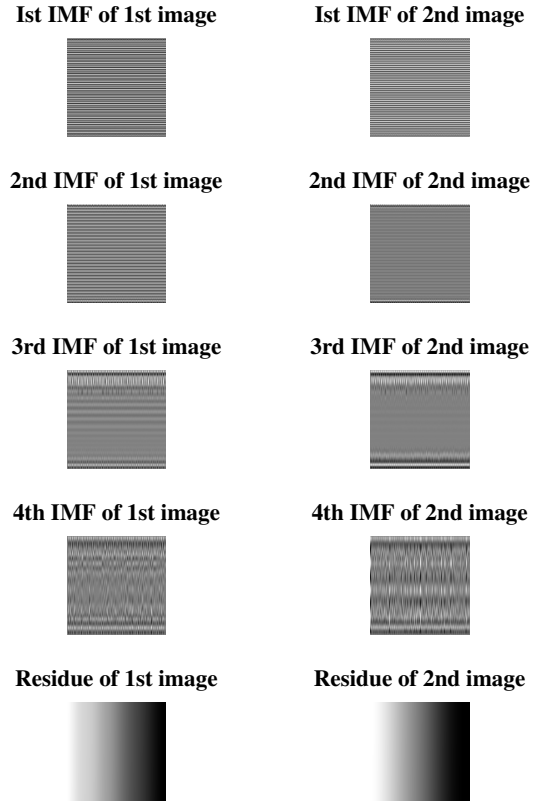


Figure 3: Decomposition of the test images using complex EMD

algorithm [6]:

$$F(m, n) = \sum_{i=1}^M [\alpha_i(m, n)A_i(m, n) + \beta_i(m, n)B_i(m, n)] \quad (9)$$

where  $(m, n)$  denotes the spatial location in the image,  $A_i$  denotes the BIMF from the first image,  $B_i$  denotes the BIMF from the second image,  $M$  denotes the number of BIMFs in the decomposition and  $\alpha_i(m, n)$  and  $\beta_i(m, n)$  are weighted coefficients which satisfy  $\alpha_i(m, n) + \beta_i(m, n) = 1$ . The values for the coefficients are determined by comparing the local variance for each scale at each location as described in [6].

It should be noted that though the complex extension of the EMD can generate the same number of IMFs with matching scales for two gray-level images, they could not guarantee the same for color images as the inter-channel (Red, Green and Blue) matching scales cannot be preserved by using complex extensions of EMD. But we can easily extend FABEMD to color image fusion by choosing the order statistics filter size as one of the

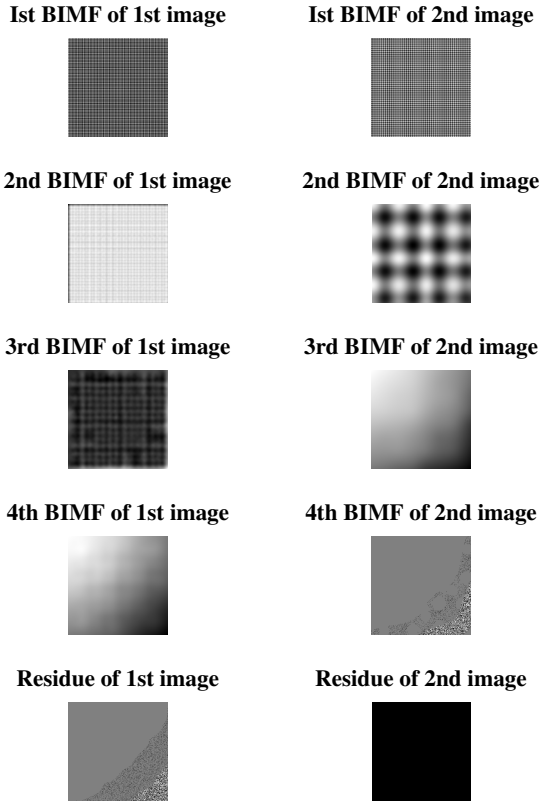


Figure 4: Decomposition of the test images using BEMD

below:

$$W_{en-g} = \min\{\min\{d_{adj-max-i-j}\}, \min\{d_{adj-min-i-j}\}\} \quad (10)$$

$$W_{en-g} = \max\{\max\{d_{adj-max-i-j}\}, \max\{d_{adj-min-i-j}\}\} \quad (11)$$

where  $d_{adj-max-i-j}$  is same as defined earlier but now  $j = 1, 2, 3$  denotes the color channel of the images used for fusion. The fusion result is demonstrated in Figure 6 and 7. From the figures, it is observed that the image edge information is contained in the high frequency scales whereas the illumination information is contained in the low frequency scales and the fused image retains all the detail of the original images as desired.

## 5 Conclusion

In this paper, the recently proposed FABEMD method is extended to the image fusion framework. As FABEMD method does not depend on surface interpolation method for envelope estimation like traditional

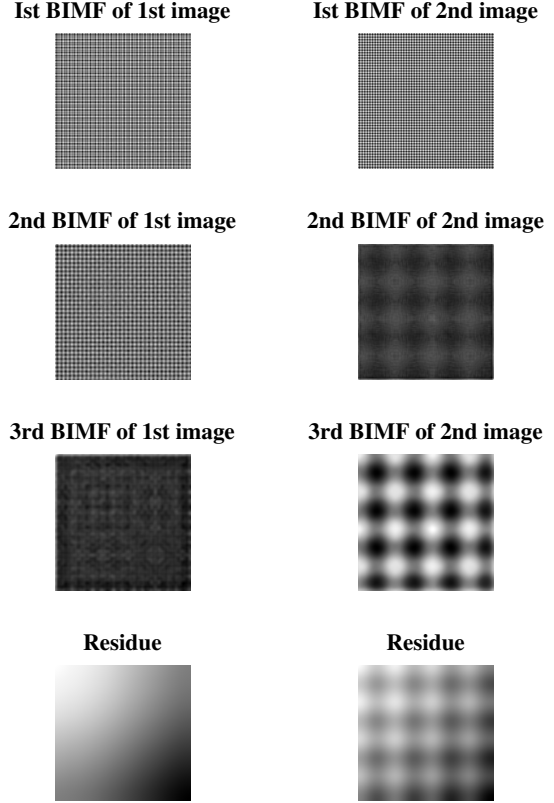


Figure 5: Decomposition of the test images using FABEMD

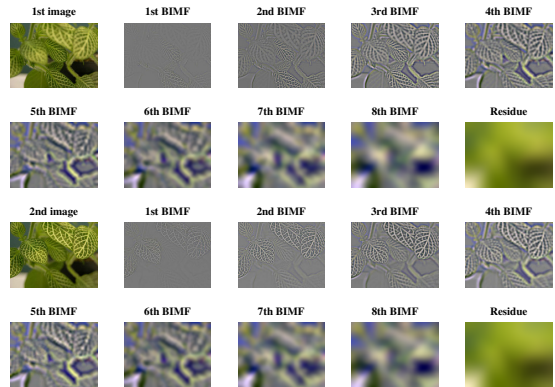


Figure 6: Partially defocused color images and their decomposition: 1st image is Background focus and the 2nd image is Foreground focus

BEMD, so the resulted BIMFs are free from inaccuracies and the overall process is much less time consuming which is a desirable feature for real-time image fusion.

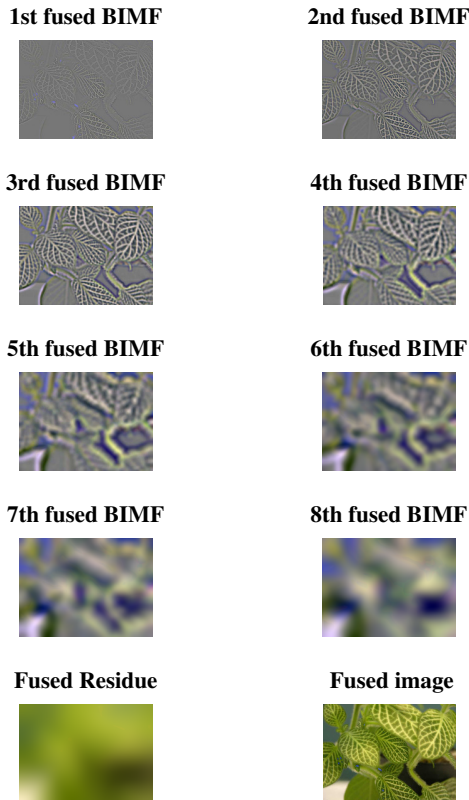


Figure 7: Fused color BIMFs and the fused all-in-focus image

Besides, it is a true two dimensional algorithm, as it considers the 2D image correlation unlike the other variants of EMD [7, 8, 9, 13]. Further, it has been shown the structure recognition capability and easily extendable to multiple image fusion as well as color image fusion as it can yield the same number of BIMFs with matching scales.

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